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ON THE TECTONIC STRESSES IN THE VICINITY OF A VALLEY AND A MOUNTAIN RANGE

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Abstract

Certain results from the theory of elasticity have been applied to an estimate of the tectonic stresses in the vicinity of a valley or a mountain range. Assuming that one has a uniform unidirectional tectonic stress state at infinity, it is shown that in a valley as well as in a mountain range, stress reversals must be expected. In a valley, these reversals occur on the shoulders, in a mountain range, at the crest. Assuming that a mountain range is built up through the action of compressional forces, the crest may be under tension and it is possible that the rift observed at the top of the Mid-Atlantic Ridge could be referring to such a stress state so that it may not be necessary to postulate an expanding Earth as has been proposed. Similarly, a valley cut into a compressional tectonic stress system (as would obtain in any mountain area) would tend to become unstable on the sides; this is precisely where serious landslides have been experienced.

Introduction

It is a well known fact that forces are present almost everywhere in the Earth's crust. On the whole, the crustal stress state is fairly uniform in any one area, but changes from area to area. A fair amount of progress has been made recently in the determination of this **ground stress** (or tectonic stress) for various areas. In particular, Lensen (1958) has shown the way by which the tectonic stress system can be ascertained from the field observation of surface faults.

On the theoretical side, it is of great importance to get an idea of how a given large-scale tectonic stress system is affected by surface irregularities such as mountain ranges, valleys and similar features (cf. Scheidegger 1961). In general, the mathematical calculation of the perturbation of a uniform (at infinity) stress state caused by the presence of surface features is a very difficult problem, so that one has to confine oneself to certain simple cases. Nevertheless, such simple cases may still represent quite reasonable models of certain natural features.

In the present paper we shall apply some results of elasticity theory which have been obtained in the literature, to corresponding surface features of the Earth. This concerns the effect of a valley or mountain range on a uniform, unidimensional (at infinity) stress state—i.e., on a large-scale tectonic stress state which is either a pure tension or a pure compression. Some of the results obtained in this fashion are rather interesting, inasmuch as it is seen, for instance, that tensional forces must be expected to arise at the crest of a mountain range, even if the latter is located in an overall compressional stress system. It has been known for some time that such reversals occur in bent thin sheets but it is believed that here is the first time that this is pointed out for the stress state in a half-space. This tension may have a connection with the rift which was discovered at the crest of the Mid-Atlantic Ridge. Similarly, if a valley is cut into a region under compression, tensile stresses occur at the shoulder. This may cause land slides.

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The elastic model

For the present analysis we shall assume that the near-surface region of the Earth can be treated as an elastic body. Although it is clear that the face of the Earth indicates that many non-elastic deformations have occurred during its history, it is held that under static conditions, the equilibrium stresses can be calculated from elasticity theory. Furthermore, over the distances under consideration, it seems reasonable to neglect the Earth's curvature.

Thus, as a basic model of the Earth's near-surface regions, we shall take an elastic half-space. On the surface of this half-space we shall then introduce irregularities. As noted in the introduction, it is extremely difficult to calculate the stresses for a general model of this type so we shall have to confine ourselves to the discussion of some simple cases. These cases refer to a semi-circular valley and a semi-circular mountain range in a unidimensional stress state. The geometry considered is therefore as shown in Fig. 1 for a valley. For a mountain range, the semicircle is simply directed upwards. The method by which such cases can be treated follows from the general principles of plane elasticity theory as they have been outlined, for instance, in the monograph by Green and Zerna (1954). It consists, in essence, in attempting to solve a bi-harmonic equation for the appropriate boundary condition. Problems of the type considered here have been solved by Maunsell (1936) and by Ling (1947); we shall apply the results obtained by these authors to the stresses in the Earth.

Stresses near a valley

We shall consider first the case of a semi-circular valley of radius a (Fig. 1). The calculations pertinent to this case have been made by Maunsell (1936). Assuming that the uniform tension at infinity be denoted by T, Maunsell calculated numerical tables of stresses (σ_{rr} = radial normal stress, $\sigma_{\theta\theta}$ = azimuthal normal

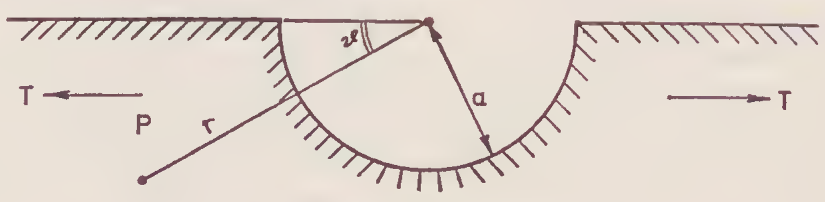

Fig. 1—Geometry of a semi-circular valley.

stress) of the valley (in Maunsell's paper, the calculations referred to notches in a plate), in a dependence of the distance a and aximuth θ from the centre of the valley (cf. Fig. 1). Maunsell's results for the present case are reproduced in Tables 1 and 2. The stress values in these Tables are in units of T. It is clear that

TABLE 1

Stresses near a semi-circular valley in dependence of the azimuth (after Maunsell 1936)

θ	$\sigma_{\theta\theta}$
5°	−0·03
10°	−0·05
15°	+0·008
20°	+0·08
30°	+0·45
40°	+0·95
50°	+1·57
60°	+2·07
70°	+2·62
80°	+2·96
90°	+3·05

TABLE 2

Stresses near a semi-circular valley in dependence on the distance r (after Maunsell 1936)

r/a	σ_{rr} for $\theta = 0$	σ_{rr} for $\theta = \pi/2$	$\sigma_{\theta\theta}$ for $\theta = \pi/2$
1·1	−0·02	+0·23	+2·51
1·25	−0·03	+0·37	+2·00
1·5	+0·09	+0·41	+1·57
1·75	+0·23	+0·39	+1·36
2·0	+0·37	+0·34	+1·24
3·0	+0·68	+0·20	+1·08
4·0	+0·81	+0·13	+1·03
5·0	+0·88	+0·09	+1·02
10·0	+0·97	+0·03	+1·002
100·0	+0·9997	+0·0003	+1·000

the calculations are independent of the sign of T; i.e., the stresses calculated by Maunsell (in terms of units of T) are independent on whether the valley is cut into an intrinsically compressional or intrinsically tensional stress state.

An inspection of Maunsell's tables shows several important facts. First of all, there is a stress concentration at the bottom of the valley ($\theta = 90°$) where $\sigma_{\theta\theta}$ is roughly three times as large as the uniform stress at infinity. This is, in fact, to be expected as the same degree of a stress concentration occurs also on a circular hole in a plate.

A second observation is that, at the shoulder of the valley, the stress has a reversed sign in comparison with the uniaxial stress at infinity. Thus, in a compressional stress state, a tensional stress occurs at the shoulder of a valley $(\theta \leqslant 10°, \frac{r}{a} = 1{\cdot}25)$. This stress reversal may have a certain significance. In mountainous areas with recent tectonic activity, the over-all stress state is generally thought to be compressional, in line with the idea that mountain building is connected with some type of crustal shortening. During the last (Pleistocene) ice age, glaciers scoured U-shaped valleys out and, from the above analysis, one would expect that a tensional stress state would then be prevalent at the 'limbs' of the U.

Rock has almost no tensional strength, and thus is would be the shoulders of a glacial valley where one would expect destruction to occur due to the presence of the over-all tectonic stress system. It may be of significance, in this connection, that commonly rock slides seem to originate high up in valleys; the Frank Slide of 1903 near the Crowsnest Pass, Alberta, Canada, is an example of this.

Stresses near a mountain range

We now turn our attention to the stresses in the vicinity of a mountain-range. We assume again that the surface of the Earth can be treated as a semi-infinite elastic body, the mountain range being represented as a semi-circular mound at right angles to the unidirectional (at infinity) stress T. The calculations pertinent to this case have been reported by Ling (1947). It is clear that, at the corners, great stress concentrations occur which are meaningless in nature, since a sharp corner of the type seen in our model will never develop. The stress concentrations will simply cause a destruction of the material. Under certain conditions this destruction can take on a startling appearance, as whole sheets of rock may be split off leaving smooth arches underneath. The theory of the origin of the arches in Yosemite Park outlined above, in essence, was originally proposed by Kieslinger (1958). The model, thus, is not a good approximation to the conditions obtaining at the point where a mountain levels off into a plain.

Thus, of interest are only the stresses that have been calculated for the summit of the mountain. Here again we see the interesting fact that a stress reversal occurs. The stress at the summit is $\sigma_{\theta\theta} = -0{\cdot}096\ T$.

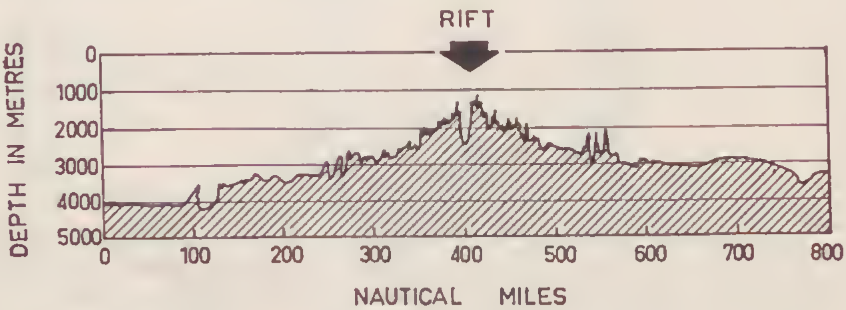

Fig. 2—Cross-section of the Mid-Atlantic Ridge, showing a rift in the centre. After Elmendorf and Heezen (1957).

This means that even in a compressional stress state, at the summit of a mountain range, the stress is tensional. This may be a significant observation. As is well known, it has been discovered that a great rift follows the crest of the Mid-Atlantic Ridge (Fig. 2). The existence of this rift has generally been taken as an indication that the Mid-Atlantic Ridge could not be caused by compressional forces like (presumably) mountain ranges on land, but rather that it is the outcome of continuous rifting (connected with an expansion of the Earth) and upwelling of magma. In the light of the above remarks, however, it would appear just as possible that the Mid-Atlantic Ridge is caused by a crustal shortening process after all. Naturally, whether tensile stresses do indeed occur at the crest of the Mid-Atlantic Ridge depends on the exact geometry of the latter. The results quoted above are only for a mountain range with a semi-circular profile. For an arch that is much flatter than a semi-circle, no tensile stresses could arise in a compressional stress system. The possible occurrence of tensile stresses at the top of a ridge in a compressional stress system is therefore pointed out here only as something that one ought to keep in mind at all times, not as a final explanation of the Mid-Atlantic Rift.

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